

Time-Continuous Bell Measurements

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We combine the concept of Bell measurements, in which two systems are projected into maximally entangled states, with the concept of continuous measurements, which concerns the evolution of a continuously monitored quantum system. For time-continuous Bell measurements based on homodyne detection of light we derive the corresponding stochastic Schrödinger equations, as well as the unconditional feedback master equations. Our results cover in particular the two scenarios of time-continuous quantum teleportation and entanglement swapping. We apply our results to show that (i) two two-level systems can be deterministically entangled via homodyne detection of light, even including photon loss approaching the fundamental limit of 50%, and (ii) a quantum state of light can be continuously teleported to a mechanical oscillator. This time-continuous remote quantum state preparation works under the same conditions as are required for optomechanical ground state cooling.

Introduction.— The quantum theory of time-continuous measurements provides profound insight into the intricate connection between decoherence, measurement and the transition from quantum to classical physics [1–12], see also [13–16]. In conjunction with continuous feedback control it builds the basis for an increasing scope of quantum physics experiments, ranging from single atoms [17–19], cavity modes [20, 21], via atomic ensembles [22–24], up to massive mechanical oscillators [25–30], demonstrating continuous measurement and control at the quantum limit.

In such experiments the system to be measured and controlled (e.g., the ion, cavity, mechanical oscillator) continuously interacts with an environment serving as a ‘meter system’ (e.g., an optical field) which in turn is continuously measured. The corresponding measurement results are fed back to the primary system as a continuous control in order to achieve the desired quantum state or dynamics. The impressive experimental progress [17–30] opens up the possibility to combine several of these setups, and to perform a continuous *nonseparable* measurement on the environments, i.e., the meter systems. Such a strategy was shown to be advantageous in the control of entanglement of parametrically interacting oscillators [31, 32]. In the extreme limit such a measurement can correspond to a continuous projection of the environments on maximally entangled states, that is, to a *continuous Bell measurement*.

In this article we introduce and study continuous Bell measurements realized via continuous homodyne detection of light [5, 6]. We derive the constitutive equations of motion—the conditional stochastic Schrödinger equation and the unconditional feedback master equation—for the monitored systems. Two scenarios are studied (Fig. 1): In the first one a single system couples to a field

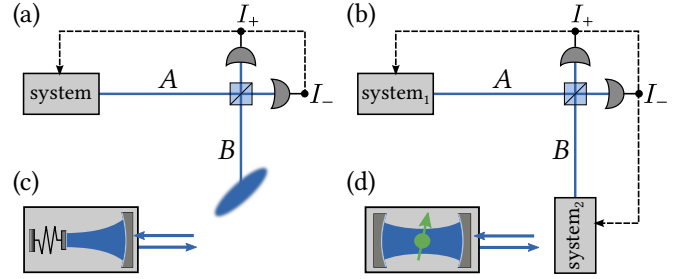


FIG. 1. Schematic setups for (a) time-continuous teleportation and (b) entanglement swapping. The systems may take the form of (c) a harmonic oscillator (e.g., an optomechanical cavity), or (d) a two-level system (e.g., a single spin).

A, which is subject to a Bell homodyne detection after interference with a second field B prepared in a general state of Gaussian (possibly squeezed) white noise. In the second setup the Bell measurement is performed on the fields emitted by two systems. Provided the system-field interactions create entanglement, the feedback master equations for the two setups cover in particular the cases of *time-continuous quantum teleportation* and *time-continuous entanglement swapping*.

We illustrate the latter process for two two-level systems, and show that they can be continuously and deterministically driven to an entangled state, ideally a Bell state. Entanglement is generated even for light losses up to 50% when the quantum capacity of the optical communication channel drops to zero [33]. This deterministic scheme for distributing long distance entanglement can provide the basis for a dissipative quantum repeater architecture [34]. Furthermore, time-continuous quantum teleportation is exemplified for an optomechanical system where the quantum state of continuous-wave light is continuously transferred to a moving mirror. We show that the experimental requirements are essentially the same as for optomechanical ground-state cooling [35, 36], the

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observation of back action noise [37], or ponderomotive squeezing [38].

Continuous Teleportation.— We consider the setup shown in Fig. 1(a): A system S couples to a 1D electromagnetic field A via a linear interaction $H_{\text{int}} = i[s a^\dagger(t) - s^\dagger a(t)]$, where s is a system operator, and the light field is described (in an interaction picture at a central frequency ω_0) by $a(t) = \int d\omega a(\omega) e^{-i(\omega - \omega_0)t}$. Analogously a second 1D field B is described by an operator $b(t)$. Our first goal is to derive a stochastic master equation (SME) for the state of the system S , conditioned on the results of a time-continuous Bell measurement on the two fields A and B . In a Markov approximation we restrict ourselves to white noise fields, which means that both $a(t)$ and $b(t)$ are δ -correlated. This allows us to introduce the Itô increment $dA(t) = a(t)dt$ (and analogously dA^\dagger , dB , dB^\dagger), and to express the time evolution of the state $|\phi\rangle$ of the overall system ($S+A+B$) as a stochastic Schrödinger equation in Itô form [15, 39],

$$d|\phi\rangle = (-iH_{\text{eff}}dt + s dA^\dagger) |\phi\rangle, \quad (1)$$

where $H_{\text{eff}} = H_{\text{sys}} - i\frac{1}{2}s^\dagger s$, with the (unspecified) system Hamiltonian H_{sys} .

We assume that the initial state of the overall system is $|\phi(0)\rangle = |\psi(0)\rangle_S |\text{vac}\rangle_A |M\rangle_B$, where $|M\rangle$ is an arbitrary pure Gaussian state defined by the eigenvalue equation $[(N + M^* + 1)b(t) - (N + M)b^\dagger(t)]|M\rangle_B = 0$. The parameters $N \in \mathbb{R}$, $M \in \mathbb{C}$ obey the relations $N \geq 0$ ($N = M = 0$ true for vacuum) and $|M|^2 = N(N + 1)$. The present white noise model essentially assumes that the squeezing bandwidth is larger than all other system time scales. Making use of the fact that $a(t)|\phi(t)\rangle = a(t)|\phi(0)\rangle = 0$ and the above eigenvalue equation, we can rewrite equation (1) in terms of the EPR operators $X_+ = (a + a^\dagger + b + b^\dagger)/\sqrt{2}$ and $P_- = -i(a - a^\dagger - b + b^\dagger)/\sqrt{2}$, which can be simultaneously measured in this setup. The resulting equation reads

$$d|\phi\rangle = [-iH_{\text{eff}}dt + s(\mu dX_+ + i\nu dP_-)] |\phi\rangle, \quad (2)$$

with $\mu = (1 - M + M^*)/(1 + N + M^*)$ and $\nu = i(1 + 2N + M + M^*)/(1 + N + M^*)$. Writing Eq. (1) in this form enables us to project (2) onto the EPR state $|I_+ I_-\rangle_{AB}$, defined by $X_+|I_+ I_-\rangle_{AB} = I_+|I_+ I_-\rangle_{AB}$ and $P_-|I_+ I_-\rangle_{AB} = I_-|I_+ I_-\rangle_{AB}$. This leads to the so-called linear stochastic Schrödinger equation [14]

$$d|\tilde{\psi}_c\rangle = \{-iH_{\text{eff}}dt + s[\mu I_+(t) + i\nu I_-(t)]dt\} |\tilde{\psi}_c\rangle, \quad (3)$$

for the unnormalized system state $|\tilde{\psi}_c\rangle$, which is conditioned on the measurement results I_\pm . Note that I_+ and I_- are real-valued, Gaussian random processes, which are proportional to the measured homodyne photocurrents. As I_\pm result from mixing the fields A and B on a beam-splitter, they carry information about both fields, and will therefore be correlated, which is one of the crucial features of a Bell measurement. We can write [6, 15]

$$I_+(t) = \sqrt{1/2}\langle s + s^\dagger \rangle_{\psi(t)} + \xi_+(t), \quad (4a)$$

$$I_-(t) = i\sqrt{1/2}\langle s - s^\dagger \rangle_{\psi(t)} + \xi_-(t), \quad (4b)$$

where $\xi_\pm(t) = dW_\pm(t)/dt$ is zero-mean, Gaussian, white noise with corresponding Wiener increments dW_\pm [40]. The (co-)variances of dW_\pm are given by

$$w_1 dt := (dW_+)^2 = [N + 1 + (M + M^*)/2]dt, \quad (5a)$$

$$w_2 dt := (dW_-)^2 = [N + 1 - (M + M^*)/2]dt, \quad (5b)$$

$$w_3 dt := dW_+ dW_- = -[i(M - M^*)/2]dt, \quad (5c)$$

as follows essentially from the initial mean values with respect to the optical fields $\langle (X_+)^2 \rangle_{\phi(0)}$, $\langle (P_-)^2 \rangle_{\phi(0)}$, etc. As expected, we in general find non-zero cross-correlations between I_+ and I_- , which depend on the squeezing properties of the input field B . Using Itô rules [40] we can construct the corresponding stochastic master equation (in Itô form) for the system state conditioned on the Bell measurement result,

$$d\rho_c = \mathcal{L}\rho_c dt + \frac{1}{\sqrt{2}}\{\mathcal{H}[\mu s]\rho_c dW_+ + \mathcal{H}[i\nu s]\rho_c dW_-\}, \quad (6)$$

where we defined $\mathcal{L}\rho = -i[H_{\text{sys}}, \rho] + \mathcal{D}[s]\rho$, the Lindblad operator $\mathcal{D}[s]\rho = (s\rho s^\dagger - \frac{1}{2}\rho s^\dagger s - \frac{1}{2}s^\dagger s\rho)$, and $\mathcal{H}[s]\rho = (s - \langle s \rangle)\rho + \rho(s - \langle s \rangle)^\dagger$.

We now apply Hamiltonian feedback proportional to the homodyne photocurrents to the system, a scenario which covers in particular the case of continuous quantum teleportation of the state of field B to the system S . We follow [41] in order to derive the corresponding unconditional feedback master equation. Hamiltonian feedback is described by a term $[\rho_c]_{\text{fb}} = \sqrt{1/2}(I_+ \mathcal{K}_+ + I_- \mathcal{K}_-)\rho_c$, where we define $\mathcal{K}_\pm \rho = -i[F_\pm, \rho]$, and Hermitian operators F_\pm . After incorporating this feedback term into the SME (6) (see supplemental information), and taking the classical average over all possible measurement outcomes, $\rho = E[\rho_c]$, we arrive at the unconditional feedback master equation

$$\begin{aligned} \dot{\rho} = & -i[H_{\text{sys}} + (1/4)\{(F_+ + iF_-)s + s^\dagger(F_+ - iF_-)\}, \rho] \\ & + (1/2)\{\mathcal{D}[s - iF_+]\rho + \mathcal{D}[s - F_-]\rho + w_3\mathcal{D}[F_+ + F_-]\rho \\ & + (w_1 - w_3 - 1)\mathcal{D}[F_+]\rho + (w_2 - w_3 - 1)\mathcal{D}[F_-]\rho\}. \end{aligned} \quad (7)$$

This is the main result of this section. The evolution of the system S thus effectively depends on the state of the field B (via w_i) which has never interacted with S , and which can in principle even change (adiabatically) in time. Eq. (7) can thus be viewed as a continuous “remote preparation” of quantum states.

In order to illustrate this point we consider the case where the target system S is a bosonic mode. For a system to be amenable to continuous teleportation the system-field interaction must enable entanglement creation. We thus set $s = c^\dagger$, with c a bosonic annihilation operator, and therefore obtain $H_{\text{int}} \propto ca(t) + c^\dagger a^\dagger(t)$, an interaction commonly known as two-mode-squeezing interaction. Additionally we choose $F_+ = i(c - c^\dagger)$ and $F_- = (c + c^\dagger)$, which means that the photocurrents I_+ , I_- will be fed-back to the x and p quadrature, respectively. The resulting equation can brought into the form

$$\dot{\rho} = -i[H_{\text{sys}}, \rho] + (2N + 1)\mathcal{D}[J]\rho, \quad (8)$$

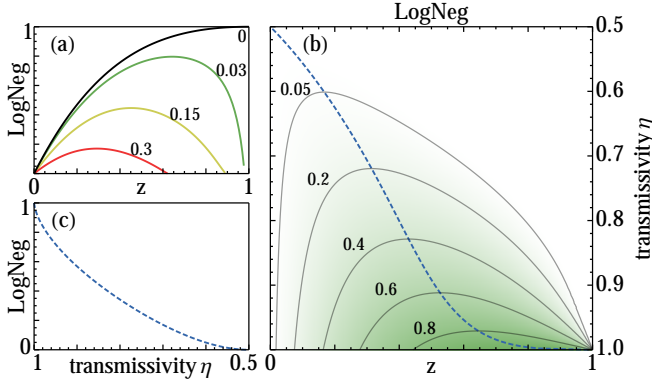


FIG. 2. (Color online) Performance of continuous entanglement swapping: (a) Entanglement (logarithmic negativity [42]) of the stationary state versus z parameterizing the light-matter interaction $\propto i[(\sqrt{z(1+z)}\sigma^+ - \sqrt{1-z}\sigma^-)a^\dagger(t) - \text{h.c.}]$, with transmissivity η as indicated. (b) Entanglement versus z and η for optimized gains. (c) Entanglement versus transmissivity for optimized z . Nonzero entanglement can be achieved even for losses approaching 50%.

where the jump operator J is determined by $J \propto -i(2N + 1 - M - M^*)x + (1 + M - M^*)p$ (with an appropriate normalization). For $H_{\text{sys}} = 0$ equation (8) has the steady-state solution $\rho_{\text{ss}} = |\psi\rangle\langle\psi|$, where $J|\psi\rangle = 0$. Up to a trivial transformation this state is identical to the input state $|M\rangle\langle M|$. Note that for the vacuum case $N = M = 0$ we find $J = c$, which means that, devoid of other decoherence terms, the system will be driven to its ground state. Below, we will come back to this scenario, and discuss its implementation on the basis of an optomechanical system in more detail. First, however, we consider the second realization of quantum dynamics conditioned on a continuous Bell measurement, namely continuous entanglement swapping, as depicted in Fig. 1(b).

Continuous Entanglement Swapping.— We now replace the Gaussian input state in mode B with a field state emitted by a second system, which couples to the field B via $H_{\text{int}} = i[s_2 b^\dagger(t) - s_2^\dagger b(t)]$. Using the same logic as before we can derive the linear stochastic Schrödinger equation for the bipartite state $|\psi\rangle$ (comprising $S_1 + S_2$)

$$d|\tilde{\psi}\rangle = [-iH_{\text{eff}}dt + s_+ I_+(t)dt + i s_- I_-(t)dt] |\tilde{\psi}\rangle, \quad (9)$$

where now $H_{\text{eff}} = H_{\text{sys}}^{(1)} + H_{\text{sys}}^{(2)} - \frac{i}{2} \sum_{i=1,2} s_i^\dagger s_i$ and $s_\pm = s_1 \pm s_2$. Accordingly, the homodyne currents read

$$I_+(t) = \sqrt{1/2} \langle s_+ + s_+^\dagger \rangle_{\psi(t)} + \xi_+(t), \quad (10a)$$

$$I_-(t) = i\sqrt{1/2} \langle s_- - s_-^\dagger \rangle_{\psi(t)} + \xi_-(t), \quad (10b)$$

and the corresponding conditional master equation is

$$d\rho_c = \mathcal{L}\rho_c dt + \sqrt{1/2} \{ \mathcal{H}[s_+] \rho_c dW_+ + \mathcal{H}[s_-] \rho_c dW_- \}, \quad (11)$$

with $\mathcal{L}\rho = -i[H_{\text{sys}}^{(1)} + H_{\text{sys}}^{(2)}, \rho] + \mathcal{D}[s_1]\rho + \mathcal{D}[s_2]\rho$. Here, the Wiener increments are uncorrelated and have unit

variance, i.e., $(dW_+)^2 = (dW_-)^2 = dt$, $dW_+ dW_- = 0$. Applying Hamiltonian feedback to either or both of the two systems in the same way as before gives rise to

$$\begin{aligned} \dot{\rho} = & -i[H, \rho] - i(1/4) [(F_+ + iF_-)s_1 + \text{h.c.}, \rho] \\ & - i(1/4) [(F_+ - iF_-)s_2 + \text{h.c.}, \rho] \\ & + (1/2) \{ \mathcal{D}[s_+ - iF_+] \rho + \mathcal{D}[s_- - F_-] \rho \}. \end{aligned} \quad (12)$$

This is the desired feedback master equation for continuous entanglement swapping. In a simple generalization of the continuous teleportation example considered above, one can check that the analogous strategy applied to two bosonic modes with $s_i = c_i^\dagger$, will indeed drive the two systems to an Einstein-Podolsky-Rosen entangled stationary state. In view of Fig. 1(b) the resulting topology comes close to a Michelson interferometer, for which a similar scheme was discussed in [43]. Note, however, that the central equations (6), (7) and (9), (12) are general, and also apply to non-Gaussian systems. As a rather surprising application we will show, that a pure entangled state of two two-level systems (TLS) can be created *deterministically*.

Consider two TLS which couple to a 1D field via operators $s_i = \sqrt{z(1+z)}\sigma_i^+ - \sqrt{1-z}\sigma_i^-$ ($i = 1, 2$; $z \in [0, 1]$). The fields are subject to a continuous Bell measurement as depicted in Fig. 1(b). The homodyne photocurrents $I_\pm(t)$ are used in a Hamiltonian feedback scheme to generate rotations of the TLS about their x and y axes according to $F_+ = G_+ \sigma_1^y + G_+ \sigma_2^y$ and $F_- = G_+ \sigma_1^x - G_- \sigma_2^x$, with gain coefficients $G_\pm = \sqrt{z/(1+z)} \left(1 \pm \sqrt{z(1+z)/(1-z)} \right)$. For this choice of s_i and F_\pm , and assuming that the levels in each TLS are degenerate (such that $H_{\text{sys}}^{(i)} = 0$), the jump operators in Eq. (12), become $J_+ = s_+ - iF_+ \propto j_1 - \lambda j_2$ and $J_- = s_- - F_- \propto j_2 + \lambda j_1$, where $j_1 = \sigma_1^- + z\sigma_2^+$ and $j_2 = \sigma_2^- + z\sigma_1^+$, and λ is a real coefficient. The common dark state of the jump operators $J_\pm |\Phi\rangle = j_{1,2} |\Phi\rangle = 0$ is the pure entangled $|\Phi\rangle \propto |00\rangle - z|11\rangle$ which becomes a maximally entangled Bell state for $z \rightarrow 1$ [34]. The particular linear combination of $j_{1,2}$ in J_\pm is chosen such, that the state $|\Phi\rangle$ is also an eigenstate of the effective Hamiltonian $\bar{H}_{\text{eff}} = \frac{1}{4} [(F_+ + iF_-)s_1 + (F_+ - iF_-)s_2 + \text{h.c.}] - \frac{i}{4} [J_+^\dagger J_+ + J_-^\dagger J_-]$ of Eq. (12), i.e., $\bar{H}_{\text{eff}} |\Phi\rangle = 0$. Together, these properties guarantee that the stationary state of Eq. (12) is the pure entangled state $|\Phi\rangle$ [44]. Note that in this way entanglement is generated deterministically, in contrast to conditional schemes based on photon counting [45–50]. Moreover, the present protocol does not require to couple nonclassical light, such as squeezed or single photon states, into cavities or to the TLS, in contrast to previous schemes [51–58].

The ideal limit of Bell state entanglement ($z \rightarrow 1$ [59]) is achieved only in the limit of infinite feedback gains G_\pm , as is to be expected for the present treatment. More sophisticated descriptions of feedback, might relieve this restriction [15]. However, in the relevant case including losses, the optimal feedback gains stay finite even

in the present description. Assuming that all passive photon losses, such as finite transmission and detector efficiency, are combined in one transmissivity/efficiency parameter η , we have to apply the generalized feedback master equation from the supplemental information instead of Eq. (12). For given η and light matter interaction, i.e., fixed z , we optimize the feedback gains G_{\pm} in order to maximize the entanglement of the stationary state ρ_{ss} . We keep the particular form of the feedback Hamiltonians F_{\pm} as it preserves the Bell diagonal structure of ρ_{ss} . Fig. 2 shows that entanglement can be achieved even for losses approaching 50%, which is where the quantum capacity of the lossy bosonic channel drops to zero [33].

The necessary strong coupling of TLS to a 1D optical field can be achieved in a variety of physical systems, such as cavities [21, 60–65] or atomic ensembles [24, 66, 67].

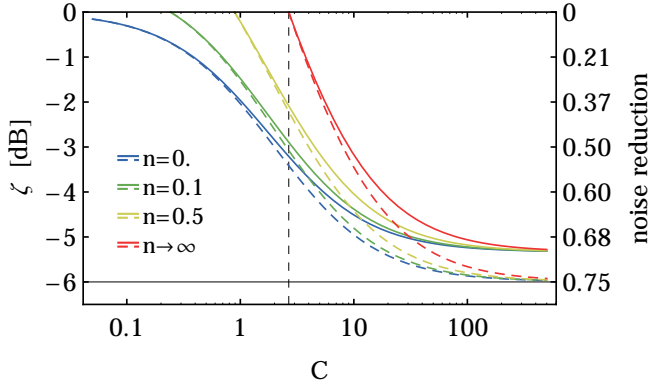


FIG. 3. (Color online) Mechanical squeezing ζ against cooperativity C for varying mechanical bath occupation $\bar{n} = 0, 1/10, 1/2, \infty$ (represented by different colors) and sideband resolution $\kappa/\omega_m = 1$ (10) [solid (dashed) lines]. The solid black line at $\zeta = -6$ dB shows the squeezing level of the input light, (corresponding to $N \approx 0.56$). The dashed vertical line shows the value of $C_{\text{crit}} = 1/[\sqrt{N(N+1)} - N] \approx 2.7$ above which mechanical squeezing is achievable for any \bar{n} .

Application to optomechanical systems.— In the remainder of this article we will show how continuous quantum teleportation can be implemented in an optomechanical system in the form of a strongly-driven Fabry–Pérot cavity with one oscillating mirror [Fig. 1(c)]. Here the system Hamiltonian (in the laser frame at ω_0) is $H_{\text{sys}} = H_0 + H_{\text{om}} = (\omega_m c_m^\dagger c_m + \Delta_c c_c^\dagger c_c) + g(c_m + c_m^\dagger)(c_c + c_c^\dagger)$, where ω_m is the mechanical frequency, $\Delta_c = \omega_c - \omega_0$ is the detuning of the driving laser (at ω_0) with respect to the cavity (at ω_c), and g is the optomechanical coupling strength. The decay rate of the cavity we denote by κ , while the coupling of the mechanical oscillator to a thermal bath (with a mean occupation number \bar{n}) we call γ . c_m and c_c are bosonic annihilation operators of the mechanical and the optical mode respectively.

In order to successfully implement continuous teleportation in this system we need to make several adaptations

to the generic case presented above. To understand this we first need a clear picture of the system’s dynamics: In the regime $g \ll \kappa \ll \omega_m$ and for $\Delta_c = -\omega_m$ the optomechanical interaction is $H_{\text{om}} \approx g(c_m c_c + c_m^\dagger c_c^\dagger)$. Under the weak-coupling condition ($g \ll \kappa$) the cavity follows the mechanical mode adiabatically, and we effectively obtain the required entangling interaction between the mirror and the outgoing field. Moreover, the mechanical oscillator resonantly scatters photons into the lower sideband at ω_c . Spectrally, the photons which are correlated with the mechanical motion are therefore located at the sideband frequency ω_c . Consequently, we have to modify the previous measurement setup in two ways: Firstly, we choose the center frequency of the squeezed input light to sit at the sideband frequency ω_c . Secondly, we now use heterodyne detection to measure quadratures on the same sideband. These two modifications, together with the adiabatic elimination of the cavity (a perturbative expansion in g/κ [8]) and a rotating-wave approximation (an effective coarse-graining in time [40]), allow us to write the SME for the mechanical system, in the rotating frame at ω_m , as

$$d\tilde{\rho}_c = \gamma_- \mathcal{D}[c_m] \tilde{\rho}_c dt + \gamma_+ \mathcal{D}[c_m^\dagger] \tilde{\rho}_c dt + \sqrt{g^2 \kappa / 2} \{ \mathcal{H}[-i\mu\eta_+ c_m^\dagger] \tilde{\rho}_c dW_+ + \mathcal{H}[\nu\eta_+ c_m^\dagger] \tilde{\rho}_c dW_- \}, \quad (13)$$

where $\eta_{\pm} = [\kappa/2 + i(\Delta_c \pm \omega_m)]^{-1}$. The first two terms describe passive cooling and heating effects via the optomechanical interaction with cooling and heating rates $\gamma_- = \gamma(\bar{n} + 1) + 2g^2 \text{Re}(\eta_-)$ and $\gamma_+ = \gamma\bar{n} + 2g^2 \text{Re}(\eta_+)$, as was derived before in the quantum theory of optomechanical sideband cooling [68, 69]. The last two terms in (13) describe the continuous measurement in the sideband resolved regime for arbitrary laser detuning Δ_c . This is an extension of the conditional master equation for optomechanical systems usually considered in the literature which concerns a resonant drive and the bad-cavity limit [10, 70] (see however [71]).

For simplicity we assume here that we can apply feedback directly to the mechanical oscillator. More realistic feedback, for example optical feedback through the cavity, will be considered elsewhere. Under this premise we can adopt the same choice of F_{\pm} as before, and arrive at a feedback master equation similar to (8), $\dot{\tilde{\rho}} = \gamma(\bar{n} + 1) \mathcal{D}[c_m] \tilde{\rho} + \gamma\bar{n} \mathcal{D}[c_m^\dagger] \tilde{\rho} + (4g^2/\kappa) \{ \lambda_1(\epsilon) \mathcal{D}[J_1(\epsilon)] + \lambda_2(\epsilon) \mathcal{D}[J_2(\epsilon)] \} \tilde{\rho}$, where $\epsilon = [1 + (4\omega_m/\kappa)^2]^{-1}$. (For details on how to derive λ_i and J_i refer to the supplemental information.) The protocol’s performance is degraded by mechanical decoherence effects and counter-rotating terms of the optomechanical coupling, which are suppressed by ϵ . For fixed input squeezing (determined by N) it is possible to describe the dynamics in terms of the mechanical bath occupation \bar{n} , the sideband resolution κ/ω_m , and the so-called cooperativity parameter $C = g^2/(\bar{n} + 1)\gamma\kappa$. To illustrate the performance of the protocol we plot the mechanical squeezing ζ , and compare it to the squeezing of the optical input state (Fig. 3). As is evident from the figure there exists a crit-

ical value $C_{\text{crit}}(N) = 1/[\sqrt{N(N+1)} - N]$ determined by the level of input squeezing, above which mechanical squeezing can be achieved for any thermal occupation \bar{n} . We emphasize that this condition on the optomechanical cooperativity is essentially the same as for the recently observed ground-state cooling [35, 36], back-action noise [37], or ponderomotive squeezing [38]!

Conclusion.— In this article we present a generalization of the standard continuous-variable Bell measurement based on homodyne detection to a continuous measurement setting. We show how this concept, together with continuous feedback, can be applied to extend existing schemes for teleportation and entanglement swapping. We specifically apply these protocols to optomechanical and two-level systems, and remarkably find, that in addition to EPR entanglement, we can also generate qubit entanglement in a deterministic fashion. We suggest that the formalism developed here can serve as a basis for continuous measurement based quantum communication and information processing with both discrete and continuous variables.

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Appendix A: Bell measurement master equations

We first treat the case of continuous teleportation. Note that due to their definition, the Itô increments commute with the unitary evolution operator at all times. It thus holds that $dA(t)|\phi(t)\rangle = dA(t)|\phi(0)\rangle = dA(t)|\text{vac}\rangle_A = 0$, and by the same reasoning $[(N + M^* + 1)dB(t) - (N + M)dB^\dagger(t)]|\phi(t)\rangle = 0$, for the initial state $|\phi(0)\rangle = |\psi(0)\rangle_S|\text{vac}\rangle_A|M\rangle_B$. Inserting these terms into (1) with appropriate prefactors yields

$$d|\phi\rangle = \{-iH_{\text{eff}}dt + s(dA^\dagger + \alpha dA + dB - \alpha dB^\dagger)\}|\phi\rangle,$$

where $\alpha = (N + M)/(N + M^* + 1)$. Rearranging this leads to Eq. (2). The probability distribution of the measurement results I_\pm is given by $\Upsilon(I_+(t), I_-(t)) = |\langle I_+ I_- |\phi(t + dt)\rangle|^2$ [6, 15], which, to first order in dt , is a Gaussian with first and second moments given by (4) and (5) respectively.

To find the SME corresponding to (3) we define $\tilde{\rho}_c(t + dt) = |\tilde{\psi}_c(t + dt)\rangle\langle\tilde{\psi}_c(t + dt)|$ and note that $|\tilde{\psi}_c(t + dt)\rangle = |\psi_c(t)\rangle + d|\tilde{\psi}_c(t)\rangle$, where $|\psi_c\rangle$ and $\tilde{\rho}_c$ are unnormalized. After normalizing $\rho_c(t + dt) = \tilde{\rho}_c(t + dt)/\text{tr}[\tilde{\rho}_c(t + dt)]$

we expand the resulting equation to second order in the noise increments dW_\pm and apply the Itô rules (5) (see [40]). With the definition $d\rho_c(t) = \rho_c(t + dt) - \rho_c(t)$ we find (6).

We follow the procedure developed in [41] to add the feedback term $[\dot{\rho}_c]_{\text{fb}} = \sqrt{1/2}(I_+K_+ + I_-K_-)\rho_c$ to the conditional master equation. Note that this term must be interpreted in the Stratonovich sense [41]. To reconcile it with equation (6) we thus have to convert (6) to Stratonovich form, add $[\dot{\rho}_c]_{\text{fb}}$, and convert to result back to Itô form. This yields

$$\begin{aligned} d\rho_c = & \mathcal{L}\rho_c dt + \frac{1}{4}[dX_+K_+ + dP_-K_-]^2\rho_c \\ & + \frac{1}{2}[dX_+K_+ + dP_-K_-]\{dW_+\mathcal{H}[\mu s] + dW_-\mathcal{H}[i\nu s]\}\rho_c \\ & + \frac{1}{\sqrt{2}}\{dW_+\mathcal{H}[\mu s] + dW_-\mathcal{H}[i\nu s]\}\rho_c \\ & + \frac{1}{\sqrt{2}}[dX_+K_+ + dP_-K_-]\rho_c, \end{aligned}$$

where the operator ordering \mathcal{KH} was used in order to get a trace-preserving master equation [41]. Using the fact that $dX_i dX_j = dX_i dW_j = dW_i dW_j$ together with (5) and taking the average with respect to the measurement outcomes $\rho = E[\rho_c]$ this equation can be brought into the form (7).

The case for entanglement swapping can be treated analogously. For the full system we can write

$$\begin{aligned} d|\phi\rangle = & [-iH_{\text{eff}}dt + s_1 dA^\dagger + s_2 dB^\dagger]|\phi\rangle \\ = & [-iH_{\text{eff}}dt + s_1(dA^\dagger + dB) + s_2(dA + dB^\dagger)]|\phi\rangle, \end{aligned}$$

which, by projection onto the EPR basis, leads to (9). Note that the initial state here is assumed to be $|\phi(0)\rangle = |\psi(0)\rangle_{S_1 S_2}|\text{vac}\rangle_A|\text{vac}\rangle_B$. The corresponding feedback equation is derived as for continuous teleportation, using the multiplication table $(dW_+)^2 = (dW_-)^2 = dt$, $dW_+ dW_- = 0$.

Appendix B: Bell measurement for non-unit detector efficiency

Passive losses due to inefficient detectors or imperfect transmission can be accounted for by introducing a combined transmissivity/efficiency $0 \leq \eta \leq 1$. The equations in the main text can be generalized in a straight-forward manner [15]. In particular we find for the case of continuous teleportation the conditional master equation

$$d\rho_c = \mathcal{L}\rho_c dt + \sqrt{\eta/2}\{\mathcal{H}[\mu s]\rho_c dW_+ + \mathcal{H}[i\nu s]\rho_c dW_-\},$$

and corresponding photocurrents

$$\begin{aligned} I_+(t) &= \sqrt{\eta/2}\langle s + s^\dagger \rangle_{\psi(t)} + \xi_+(t), \\ I_-(t) &= i\sqrt{\eta/2}\langle s - s^\dagger \rangle_{\psi(t)} + \xi_-(t). \end{aligned}$$

Including feedback as $[\dot{\rho}_c]_{\text{fb}} = \sqrt{1/2\eta} (I_+ \mathcal{K}_+ + I_- \mathcal{K}_-) \rho_c$ gives rise to the feedback master equation

$$\begin{aligned} \dot{\rho} = & -i [H_{\text{sys}} + (1/4) \{ (F_+ + iF_-)s + s^\dagger(F_+ - iF_-) \}, \rho] \\ & + \frac{1}{2} \left\{ \mathcal{D}[s - iF_+] \rho + \mathcal{D}[s - F_-] + \frac{w_3}{\eta} \mathcal{D}[F_+ + F_-] \right. \\ & \left. + \frac{w_1 - w_3 - \eta}{\eta} \mathcal{D}[F_+] + \frac{w_2 - w_3 - \eta}{\eta} \mathcal{D}[F_-] \right\} \rho. \end{aligned}$$

Applying the same considerations to the case of entanglement swapping leads to

$$d\rho_c = \mathcal{L}\rho_c dt + \sqrt{\eta/2} \{ \mathcal{H}[s_+] \rho_c dW_+ + \mathcal{H}[s_-] \rho_c dW_- \},$$

and

$$\begin{aligned} \dot{\rho} = & -i [H, \rho] - i(1/4) [(F_+ + iF_-)s_1 + \text{h.c.}, \rho] \\ & - i(1/4) [(F_+ - iF_-)s_2 + \text{h.c.}, \rho] \\ & + (1/2) \{ \mathcal{D}[s_+ - iF_+] \rho + \mathcal{D}[s_- - F_-] \rho \} \\ & + (1 - \eta)/\eta \{ \mathcal{D}[F_+] \rho + \mathcal{D}[F_-] \rho \}, \end{aligned}$$

replacing Eq. (11) and (12) respectively.

Appendix C: Diagonalization of non-Lindblad terms

In general the feedback master equations (7) and (12) are not in Lindblad form as the prefactors of the operators \mathcal{D} can be negative. To cure this we can rewrite the non-unitary part of the evolution in terms of $R = (x, p)^T$ as $\dot{\rho} = \sum_{ij} \Lambda_{ij} (R_i \rho R_j - \frac{1}{2} \rho R_j R_i - \frac{1}{2} R_j R_i \rho)$, where Λ is a Hermitian matrix. By virtue of the eigenvalue decomposition of Λ we can write $\dot{\rho} = \sum_i \lambda_i \mathcal{D}[J_i] \rho$ with $J_i = v_i \cdot R$, where λ_i and v_i ($i = 1, 2$) are the eigenvalues and eigenvectors of Λ respectively.

Appendix D: Continuous optomechanical teleportation

To derive Eq. (13) we start from the SME describing heterodyne detection of the optomechanical cavity's output light. It can be obtained from equation (6) by replacing $s \rightarrow c_c e^{i\Delta_{\text{lo}} t}$, where Δ_{lo} is the local oscillator detuning, and adding decoherence terms for the mechanical subsystem, which are due to its coupling to a thermal bath. After going into an interaction picture with $H_0 = \omega_m c_m^\dagger c_m + \Delta_c c_c^\dagger c_c$ we can adiabatically eliminate the cavity mode, by expanding the equations in the small parameter g/κ up to first order [8, 72]. Under this approximation and after setting $\Delta_{\text{lo}} = \omega_m$ the SME takes the form

$$\begin{aligned} d\rho_c = & \mathcal{L}_c \rho_c dt - g^2 [c_m + c_m^\dagger, y \rho_c - \rho_c y^\dagger] dt + \\ & \sqrt{g^2 \kappa/2} \{ \mathcal{H}[-i\mu e^{i\omega_m t} y] \rho_c dW_+ + \mathcal{H}[\nu e^{i\omega_m t} y] \rho_c dW_- \} \end{aligned}$$

where $\mathcal{L}_m \rho = -i[\omega_m c_m^\dagger c_m, \rho] + \gamma(\bar{n}+1)\mathcal{D}[c_m] \rho + \gamma\bar{n}\mathcal{D}[c_m^\dagger] \rho$, and $y = \eta_- c_m + \eta_+ c_m^\dagger$. The steady-state mean amplitude of the intracavity field is $\langle c_c \rangle = -ig\langle y \rangle$. The heterodyne photocurrents can thus be obtained from (4) by the replacement $\langle s \rangle \rightarrow -ig\langle y \rangle e^{i\omega_m t}$, thus

$$\begin{aligned} I_+ = & -i\sqrt{g^2 \kappa/2} \langle y e^{i\omega_m t} - \text{h.c.} \rangle(t) + \xi_+(t) \\ I_- = & \sqrt{g^2 \kappa/2} \langle y e^{i\omega_m t} + \text{h.c.} \rangle(t) + \xi_-(t) \end{aligned}$$

To apply the rotating wave approximation we go to the rotating frame of the mechanical oscillator (in the following denoted by $\tilde{\rho}_c$) [73] and take a time average over an interval δt , which comprises many mechanical periods, but is short on the system timescales in the rotating frame, i.e., $1/g \gg \delta t \gg 1/\omega_m$. Under this assumption we can pull $\tilde{\rho}_c$ out from under the integral and drop terms rotating at a frequency ω_m . At the same time we treat δt as infinitesimal on the system's timescale and therefore replace $\delta t \rightarrow dt$. This leaves us with the equation

$$\begin{aligned} d\tilde{\rho}_c = & \gamma_- \mathcal{D}[c_m] \tilde{\rho}_c dt + \gamma_+ \mathcal{D}[c_m^\dagger] \tilde{\rho}_c dt \\ & + \sqrt{g^2 \kappa/2} \{ \mathcal{H}[-i\mu\eta_+ c_m^\dagger] \tilde{\rho}_c dW_{0+} + \mathcal{H}[\nu\eta_+ c_m^\dagger] \tilde{\rho}_c dW_{0-} \}. \end{aligned}$$

We also introduced the time-averaged Wiener increments $dW_{0\pm} = \int dW_\pm$, which approximately fulfill (5), as far as $\tilde{\rho}_c$ is concerned. In principle this equation contains additional measurement terms corresponding to sideband modes at frequencies $\pm 2\omega_m$, which in a RWA are not correlated to the DC modes $dW_{0\pm}$ and were therefore neglected. By renaming $dW_{0\pm} \rightarrow dW_\pm$ we arrive at (13). Note that by applying the RWA to the decoherence and the measurement terms consistently we assure that the resulting equation is a valid Belavkin equation [74].

To do feedback we do the same coarse-graining procedure for the photocurrents $I_{0\pm} = \int I_\pm dt$, and find

$$\begin{aligned} I_{0+} = & -i\sqrt{g^2 \kappa/2} \langle \eta_+ c_m^\dagger - \text{h.c.} \rangle(t) + \xi_+(t), \\ I_{0-} = & \sqrt{g^2 \kappa/2} \langle \eta_+ c_m^\dagger + \text{h.c.} \rangle(t) + \xi_-(t). \end{aligned}$$

Within these approximations the system is equivalent to the generic case in the main text. Applying the same feedback procedure we obtain

$$\begin{aligned} \dot{\tilde{\rho}}_m = & \{ \gamma(\bar{n}+1)\mathcal{D}[c_m] + \gamma\bar{n}\mathcal{D}[c_m^\dagger] \} \tilde{\rho}_m \\ & + (4g^2/\kappa) \{ (1+\epsilon)\mathcal{D}[c_m] + w_3\mathcal{D}[x_m + p_m] \\ & + (w_1 - w_3 - 1)\mathcal{D}[p_m] + (w_2 - w_3 - 1)\mathcal{D}[x_m] \} \tilde{\rho}_m, \end{aligned}$$

In view of the previous section we can diagonalize this equation to obtain

$$\begin{aligned} \dot{\tilde{\rho}} = & \{ \gamma(\bar{n}+1)\mathcal{D}[c_m] + \gamma\bar{n}\mathcal{D}[c_m^\dagger] \} \tilde{\rho} \\ & + (4g^2/\kappa) \{ \lambda_1(\epsilon)\mathcal{D}[J_1(\epsilon)] + \lambda_2(\epsilon)\mathcal{D}[J_2(\epsilon)] \} \tilde{\rho}, \quad (\text{D1}) \end{aligned}$$

where λ_i and J_i are obtained from the eigenvalue decomposition of

$$\Lambda = \begin{pmatrix} w_2 - \frac{1}{2}(1+\epsilon) & -w_3 + \frac{1}{2}(1+\epsilon) \\ -w_3 - \frac{1}{2}(1+\epsilon) & w_1 - \frac{1}{2}(1+\epsilon) \end{pmatrix},$$

where $\epsilon = [1 + (4\omega_m/\kappa)^2]^{-1}$. In the limit $\epsilon \rightarrow 0$ Eq. (D1) reduces to (8), apart from the decoherence terms of the mechanical subsystem, which counteract the squeezing of the mechanical mode by driving it towards a thermal

state. Operating the protocol in a regime of strong cooperativity $g^2/\kappa \gg \gamma(\bar{n} + 1)$ suppresses these perturbative effects.

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